Student Number:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2018

MATHEMATICS

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black pen.
- Board-approved calculators & templates may be used.
- A reference sheet is provided.
- In Questions 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work.

Total Marks: 100

Section I: 10 marks

- Attempt Questions 1 – 10.
- Answer on the multiple choice sheet provided.
- Allow about 15 minutes for this section.

Section II: 90 Marks

- Attempt Question 11 - 16
- Answer on lined paper provided. Start a new page for each new question.
- Allow about 2 hours & 45 minutes for this section.

The answers to all questions are to be returned in separate, *stapled* bundles, clearly labelled as Question 11, Question 12, etc. Each answer sheet must show your *candidate number*.

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 An arc of a circle of radius 0.1 units subtends an angle of $15°$. Which of the following gives the length of that arc, correct to three decimal places?

A. 1.500

B. 0.262

C. 0.026

D. 0.008

2 Which expression is a factorisation of $8x^3 + 27$?

A. $(2x-3)(4x^2+12x-9)$

B. $(2x+3)(4x^2-12x+9)$

C. $(2x-3)(4x^2+6x-9)$

D. $(2x+3)(4x^2-6x+9)$

3 The complete solution to $|2x-1| = 3x$ is which of the following?

A. $x=-1$

B. $x=-1,\ x=\dfrac{1}{5}$

C. $x=\dfrac{1}{5}$

D. $x=1$

4 Which inequality gives the domain of the function $f(x) = \dfrac{1}{\sqrt{x+5}}$?

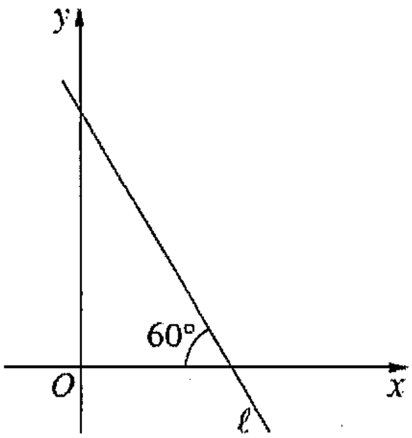
A. $x > -5$

B. $x \geq -5$

C. $x < -5$

D. $x \leq -5$

5 The diagram below shows the line ℓ.



What is the slope of the line ℓ?

A. $-\dfrac{1}{\sqrt{3}}$

B. $\dfrac{1}{\sqrt{3}}$

C. $-\sqrt{3}$

D. $\sqrt{3}$

6 Which equation represents the line perpendicular to $2x - 3y = 8$, passing through the point $(2, 0)$?

A. $3x + 2y = 4$

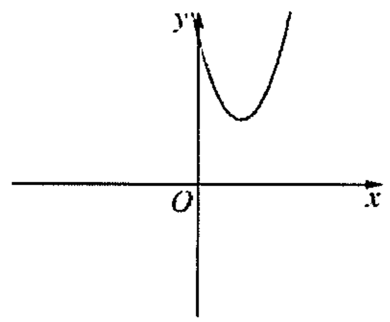
B. $3x + 2y = 6$

C. $3x - 2y = -4$

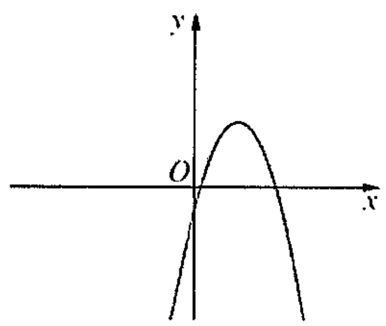
D. $3x - 2y = 6$

7 Which diagram best shows the graph of the parabola $y = 3 - (x-2)^2$?

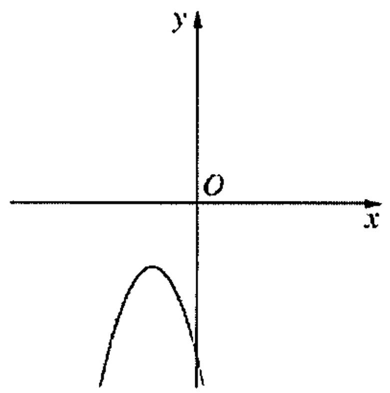
(A)



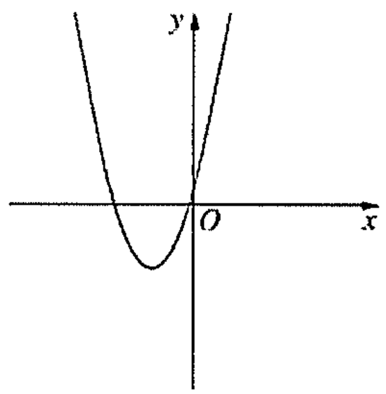
(B)



(C)



(D)



8 A parabola has focus $(5, 0)$ and directrix $x = 1$. Which of the following is the equation of the parabola?

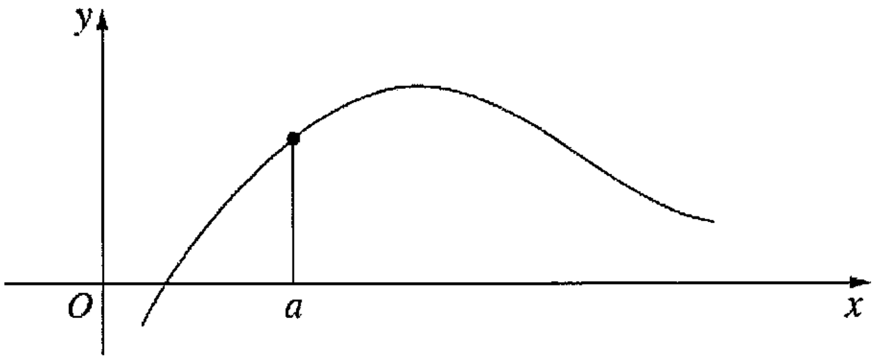
A. $y^2 = -16(x-5)$

B. $y^2 = -8(x-3)$

C. $y^2 = 8(x-3)$

D. $y^2 = 16(x-5)$

9 The graph of $y = f(x)$ is shown in the diagram below.



Which of the following statements is true?

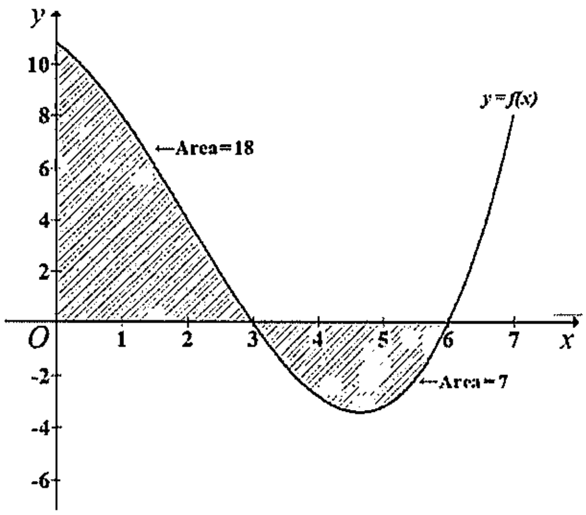
A. $f'(a) > 0$ and $f''(a) < 0$

B. $f'(a) > 0$ and $f''(a) > 0$

C. $f'(a) < 0$ and $f''(a) < 0$

D. $f'(a) < 0$ and $f''(a) > 0$

10 The diagram below shows the areas of two regions bounded by the graph of $y = f(x)$ and the x-axis. Which of the following gives the value of the integral $\int_0^6 \frac{f(x)}{3}\,dx$?



A. $\int_0^6 \frac{f(x)}{3}\,dx = \frac{11}{3}$ B. $\int_0^6 \frac{f(x)}{3}\,dx = \frac{25}{3}$ C. $\int_0^6 \frac{f(x)}{3}\,dx = 33$ D. $\int_0^6 \frac{f(x)}{3}\,dx = 75$

Section II

90 marks

Attempt Questions 11–16

Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/ or calculations.

Question 11 (15 Marks) Start a new sheet of paper.

(a) Find the value of $2^{-1.2}$ correct to two significant figures. **1**

(b) Express $\dfrac{4}{\sqrt{5}+2}$ with a rational denominator. **2**

(c) Simplify $\dfrac{x}{3}+\dfrac{3x-1}{2}$. **2**

(d) Solve the simultaneous equations **2**

$$x+y=1$$
$$2x-y=5$$

(e) Solve $\sin x = -\dfrac{1}{\sqrt{2}}$ for $0 \le x \le 2\pi$. **2**

(f) Find the exact value of $\sin\dfrac{\pi}{4}+\sin\dfrac{2\pi}{3}$. **2**

(g) Find the equation of the tangent to the curve $y=2e^x$ at the point where $x=1$. **2**

(h) Find a primitive function of x^2+7. **2**

End of Question 11

Question 12 (15 Marks) Start a new sheet of paper.

(a) In an arithmetic series, the eighth term is $T_8 = 150$ and the sum of the first seven terms is $S_7 = 546$. Find the value of the sixteenth term. **3**

(b) Differentiate the following functions with respect to x:

(i) $(3x^2 + 4)^7$ **2**

(ii) xe^{-x} **2**

(iii) $\dfrac{\tan x}{x}$ **2**

(c) Evaluate the following integrals:

(i) $\displaystyle\int_0^{\pi/4} 4\sin 2x\,dx$ **2**

(ii) $\displaystyle\int_{-1}^{1} e^{2x}\,dx$ **2**

(d) Find $\displaystyle\int \frac{x}{x^2+3}\,dx$. **2**

End of Question 12

Question 13 (15 Marks) Start a new sheet of paper.

(a) A box contains seven cards, with each card labelled with one of the following numbers:

0, 5, 5, 9, 9, 9.

A person draws one card at random from the box, and then draws a second card at random *without* replacing the first card.

(i) What is the probability that the person draws a *9*, then a *5*? **2**

(ii) What is the probability that the sum of the two numbers drawn is at most 10? **1**

(iii) What is the probability that the second card drawn is labelled *5*? **2**

(b) Consider the curve $y = 7 + 4x^3 - 3x^4$.

(i) Find the co-ordinates of the stationary points **2**

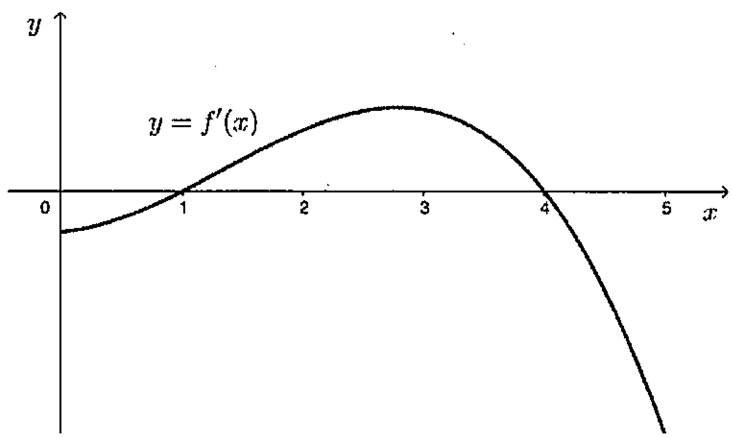
(ii) Determine the nature of the stationary points. **2**

(iii) Find all points of inflexion. **2**

(iv) Sketch the curve over the domain $-1 \le x \le 2$. **2**

Note: you need not find any x-intercepts that have not already been found.

(c) **2**



The diagram above shows the graph of the gradient function $f'(x)$ of the curve $y = f(x)$. For what values of x does $f(x)$ have a local maximum? Justify your answer.

End of Question 13

Question 14 (15 Marks) Start a new sheet of paper.

(a) The mass m grams of a dissolving tablet present after t seconds is given by

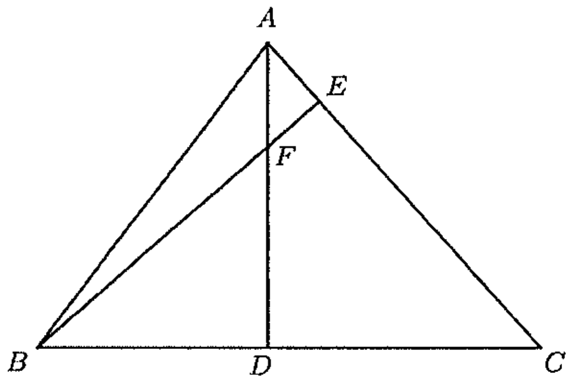
$$m = 300e^{-kt}$$

where k is a positive constant. At seven seconds, the tablet has dissolved to one third of its original mass.

(i) Find the value of k, correct to three significant figures. **2**

(ii) How long, to the nearest second, will it take for the tablet to dissolve to at least 1% of its initial mass? **2**

(b) **3**



In the diagram above, $AD \perp BC$ and $BE \perp AC$. If $BE = 11$, $AD = 9$ and $CD = 8$, find the length of CE, ensuring you give all necessary reasoning.

(c) A particle moves along a line and has position x metres at time t seconds given by

$$x = 2\cos t - t, \qquad t \geq 0.$$

Assume that left of origin is negative, right of origin is positive.

(i) Where is the particle initially? **1**

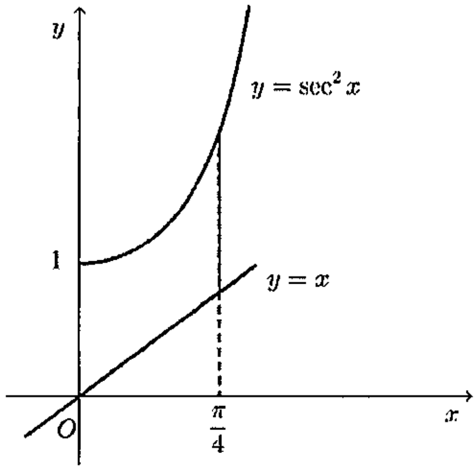
(ii) Find an expression for the velocity of the particle. **1**

(iii) In what direction is the particle moving when $t = 0$ (toward, or away from, the origin)? **1**

(iv) Determine the time at which the particle first comes to rest. **2**

Question 14 continues on page 9

(d) **3**



Find the area bounded by the curves $y = \sec^2 x$, $y = x$ and the lines $x = 0$ and $x = \dfrac{\pi}{4}$.
Leave your answer in exact form.

End of Question 14

Question 15 (15 Marks) Start a new sheet of paper.

(a) Let α and β be the roots of the quadratic equation $x^2 + 4x + 1 = 0$.

(i) State $\alpha + \beta$ and $\alpha\beta$. **1**

(ii) Hence find $(\alpha - \beta)^2$ **2**

(b) Che takes a loan of \$500 000 at an interest rate of 9% per annum, compounded monthly, to be charged on the outstanding balance. The loan is to be repaid in equal monthly installments of \$$M$ over a 25 year period.

(i) If interest is charged on the balance of the loan at the end of the month, just before the monthly repayment is made, show that the amount owing on the loan after three months A_3 is given by **2**

$$A_3 = \$500\,000(1.0075)^3 - M[1 + 1.0075 + 1.0075^2]$$

(ii) Assuming that the pattern indicated in (i) holds, the amount owing after n months may be given by **2**

$$A_n = \$500\,000(1.0075)^n - M[1 + 1.0075 + 1.0075^2 + \cdots + 1.0075^{n-1}]$$

How much should Che pay per month in order to have the loan paid off by the end of the 25th year?

(iii) What is the total interest paid on this loan? **1**

(c) A tank that initially contains 16 000 litres of petroleum is to be drained. At t minutes, the rate at which the volume V of petroleum decreases is given by

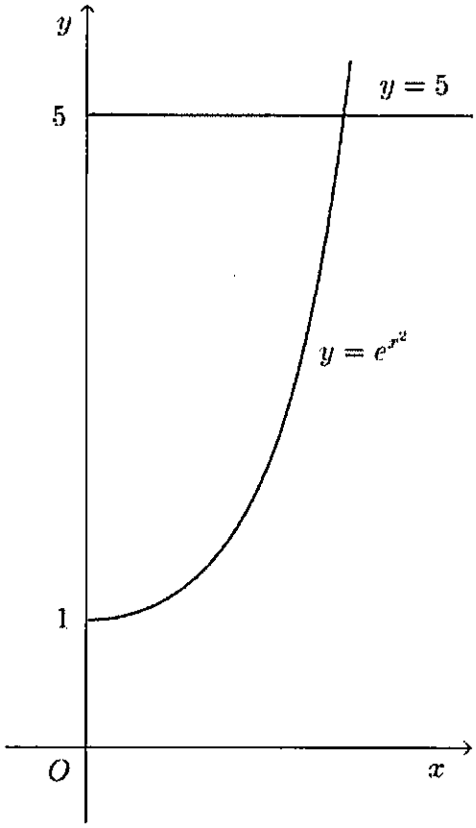
$$\frac{dV}{dt} = -40(30 - t)$$

(i) Find $V(t)$, the volume of petroleum remaining in the tank at time t. **2**

(ii) How long will it take to empty the tank? **1**

Question 15 continues on page 11

(d)



The area bounded by the y-axis, the line $y = 5$ and the curve $y = e^{x^2}$ is rotated about the y-axis to form a solid of revolution.

(i) Show that $V = \pi \int_1^5 \log_e y \, dy$. **2**

(ii) Copy and complete the table below, giving your entries to three decimal places: **2**

y	1	3	5
$\log_e y$			

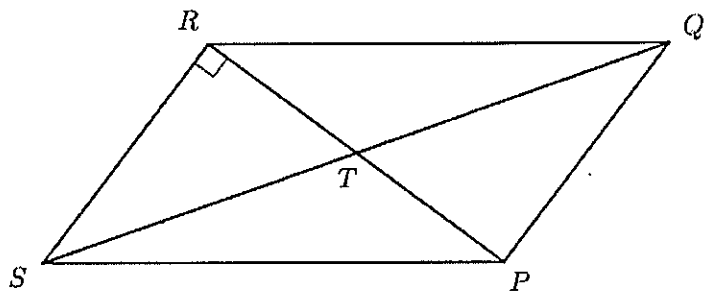
Using Simpson's Rule, approximate the volume of the solid of revolution. Give your answer correct to three decimal places.

End of Question 15

Question 16 (15 Marks) Start a new sheet of paper.

(a) Solve $\log_4(x+1) = \log_2 x$ where $x > 0$. **2**

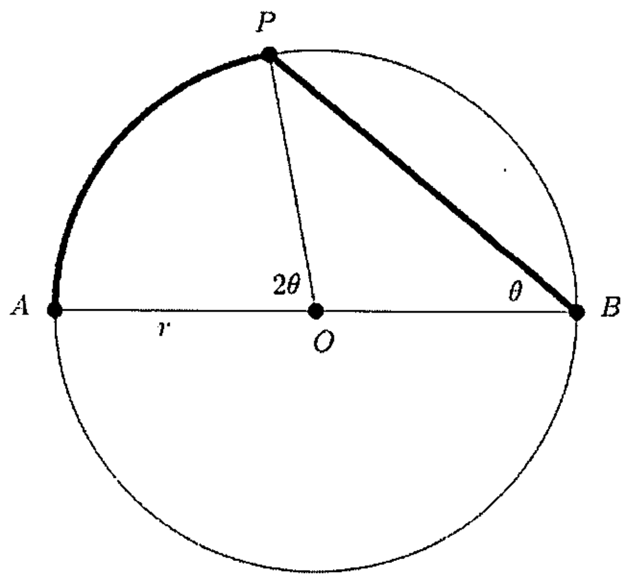
(b) The quadrilateral $PQRS$ is a parallelogram with PR perpendicular to RS. The two diagonals intersect at T. **3**



Show that $PS^2 = ST^2 + 3PT^2$.

Question 16 continues on page 13

(c)



The diagram above shows a circular lake of radius r km and diameter AB. O is the centre of the circle, and you may assume that $\angle POA = 2\angle PBO$ always and $0 \leq \theta \leq \frac{\pi}{2}$.

A person wishes to move from A to B and can do so via a combination of walking along the shore (along a circular arc) and rowing across the lake (in a straight line). The person can walk at a constant speed of 4 km/h and row at a constant speed of 2 km/h.

(i) Show that the time t_1 taken to walk along the shore AP is **2**

$$t_1 = \frac{r\theta}{2}$$

(ii) Given $\cos 2\theta = 2\cos^2\theta - 1$ or $\sin 2\theta = 2\sin\theta\cos\theta$, show that the time t_2 taken to row across the lake PB is **3**

$$t_2 = r\cos\theta$$

ensuring that you justify your result by considering the given domain.

(iii) Hence find, in terms of r:

(α) the maximum time taken for the journey; **3**

(β) the minimum time taken for the journey. **2**

End of Paper

Q11 2018 2u TRIAL $7 + 4 + 4$

a-d ef gh

7

a) $2^{-1.2} = 0.43527\ldots = 0.44$ (2 sf) 1

b) $\frac{4}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{4(\sqrt{5}-2)}{1} = 4\sqrt{5}-8$ 1 1

c) $\frac{2x+9x-3}{6} = \frac{11x-3}{6}$ 1 1 or $\frac{11x}{6} - \frac{1}{2}$

d) $x+y=1$

$2x-y=5$

$3x=6$

$\therefore x=2, \ y=-1$ 1+1

e) $\sin x = -\frac{1}{\sqrt{2}} \quad 0 \le x \le 2\pi$

4

$x = \frac{5\pi}{4}$ or $\frac{7\pi}{4}$ 1+1

Answer in 225° 315° only 1 mark.

f) $\sin\frac{\pi}{4} + \sin\frac{2\pi}{3}$

$= \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}$ 1 1

or $= \frac{\sqrt{2}+\sqrt{3}}{2}$

g) $y = 2e^x \quad y' = 2e^x\Big]_{x=1} = 2e$ 1

$x=1, \ y=2e$

4

$y - 2e = 2e(x-1)$

$y - \cancel{2e} = 2ex - \cancel{2e}$

$y = 2ex$ 1

h) $f'(x) = x^2+7$ 1

$f(x) = \frac{x^3}{3} + 7x + k$ 1 or $\frac{x^3}{3} + 7x$

Q12 Yr 12 TRIAL 2u 2018

a) Given A.S $T_n = a + (n-1)d$ $S_n = \frac{a+l}{2}.n$

$T_8 = a + 7d = 150$

$S_n = \frac{a + a + (n-1)d}{2}$

3 $S_7 = 546$

$S_7 = \left(\frac{2a+6d}{2}\right) \times 7 = 7(a + 3d) = 546$ $S_n = \frac{2a + (n-1)d}{2}.n$

$a + 3d = 78$

$a + 7d = 150$
$a + 3d = 78$

$4d = 72$
$d = 18$ / $a = 150 - 7 \times 18 = 24$ /

$T_{16} = a + 15d = 24 + 15 \times 18$

$T_{16} = \underline{\underline{294}}$ /

b) i) $y = (3x^2+4)^7$

12 $y' = 7(3x^2+4)^6(6x)$ 1+1

$= \underline{42x(3x^2+4)^6}$

ii) $y = xe^{-x}$

$y' = -xe^{-x} + e^{-x} = \underline{e^{-x}(1-x)}$ 1 + 1

iii) $y = \frac{\tan x}{x}$

$y' = \frac{x\sec^2 x - \tan x}{x^2}$ 1 1 or $\frac{x(\tan^2 x+1) - \tan x}{x^2}$

$y' = \frac{\tan^2 x + 1}{x} - \frac{\tan x}{x^2}$ #.

c i) $\int_0^{\frac{\pi}{4}} 4 \sin 2x \, dx$

$= -\frac{4 \cos 2x}{2}\Big]_0^{\frac{\pi}{4}}$ 1

$= -2 \left(\cancel{\cos \frac{\pi}{2}} - \cancel{\cos 0} \right)$

$= \underline{\underline{2}}$ 1

ii) $\int_{-1}^{1} e^{2x} dx = \frac{e^{2x}}{2}\Big]_{-1}^{1} = \underline{\underline{\frac{1}{2}(e^{2} - e^{-2})}}$ 1

d) $\frac{1}{2}\int \frac{2x \, dx}{x^2+3}$ 1

$= \underline{\underline{\frac{1}{2} \ln (x^2+3) + c}}$ 1

1m only w/o c

Trials 2018

MATHEMATICS: Question 13 — 1/3

Suggested Solutions	Marks Awarded	Marker's Comments
a) i) 0 5 5 9 9 9		mistake
$P(9 \text{ then } 5) = \frac{3}{6} \times \frac{2}{5}$	1	* $\frac{5}{30} = \frac{1}{6}$
$= \frac{6}{30} = \frac{1}{5}$	1	
ii) P (at most 10)		unfortunately correct answer ONLY!
$\frac{12}{30} = \frac{2}{5}$	1	
iii) P (2nd 5)		
$P(5\ 5)$ or $P(\bar{5}\ 5)$		
$= \frac{2}{6} \times \frac{1}{5} + \frac{4}{6} \times \frac{2}{5}$	1	
$= \frac{2}{30} + \frac{8}{30} = \frac{1}{3}$	1	

Table for ii):

	0	5	5	9	9	9
0	X	5 ✓	5 ✓	9 ✓	9 ✓	9 ✓
5	5 ✓	X	10 ✓	14	14	14
5	5 ✓	5 ✓	X	14	14	14
9	9 ✓	14	14	X	18	18
9	9 ✓	14	14	18	X	18
9	9 ✓	14	14	18	18	X

MATHEMATICS: Question 13 2/3

Suggested Solutions	Marks Awarded	Marker's Comments

b) i) $y = 7 + 4x^3 - 3x^4$

$= 12x^2 - 12x^3$

$= 12x^2(1-x)$

$\therefore x = 0, \ y = 7$

$x = 1, \ y = 8.$

$\therefore$ coordinates of stationary points $(0, 7)$, $(1, 8)$

1 — for $(0, 7)$

1 — for $(1, 8)$

ii)

x	<0	0	$\frac{1}{2}$	1	>1
y'	+	•	+	•	–
grad	/	—	/	—	\

horizontal POI at $(0, 7)$

relative max / local max at $(1, 8)$

• stationary pts

+ +ve value

– –ve value.

1 — some sort of table / graph

1 describing both stationary pts.

iii) $\frac{dy}{dx} = 12x^2 - 12x^3$

$\frac{d^2y}{dx^2} = 24x - 36x^2$

when $\frac{d^2y}{dx^2} = 0 \quad x = 0$ or $\frac{2}{3}$

x	$\frac{1}{2}$	$\frac{2}{3}$	1
$d''y$	3	0	-12
concavity	$\cup$	∘	$\cap$

1

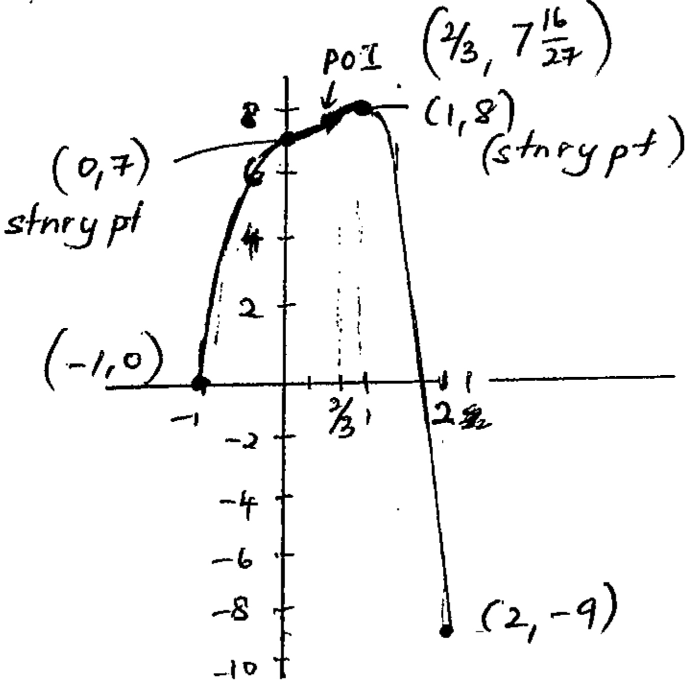
POI at $(0, 7)$ and $\left(\frac{2}{3}, \frac{205}{27}\right)$

or $7\frac{16}{27}$.

1

Suggested Solutions	Marks Awarded	Marker's Comments

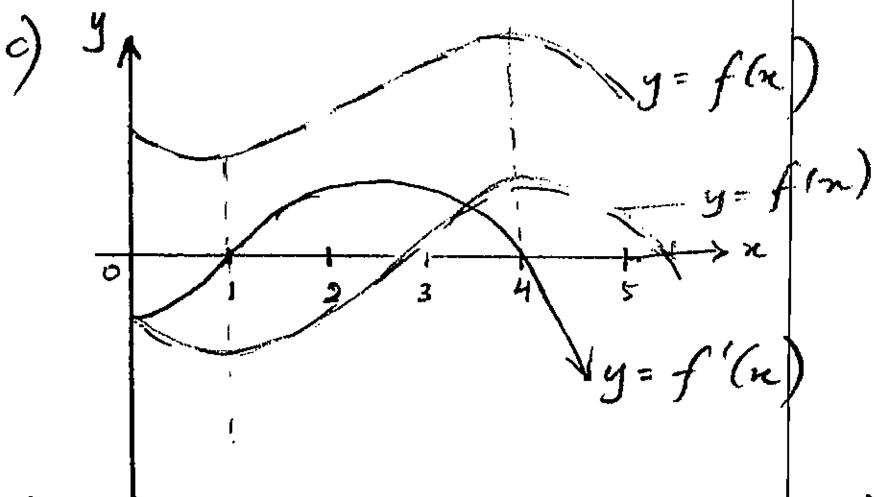
iv) Domain $-1 \leq x \leq 2$



1 — shape

1 — all pts including $(-1,0)$ and $(2,-9)$

c)



1 — justify with table or graph or explaination

1 — for stating local ~~max~~ max at $x=4$

OR local max on $f(x)$ is at S.P. $f'(x)$ changes from +ve to ~~+ve~~ -ve at SP.

x	0	1	2	3	4	5
$f'(x)$	\	—	/	/	—	\

$f''(x) < 0$ at $x = 4$

Explaination: $f'(x)$ graph SP at $x = 1$ or $x = 4$

graph is concave up at $x = 1$ $\therefore$ local min

$f''(x) < 0$ at $x=4$ $f(x)$ has local max at $x = 4$

Trials 2018

MATHEMATICS: Question...14...		1/4
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
a) i) $m = 300e^{-kt}$, $k > 0$ $t = 7 \therefore m = 100$ $\frac{1}{3} = e^{-7k}$ $k = \frac{\ln 3}{7}$ ≈ 0.157	1 1	
ii) 1% of initial mass $= 3g$. $3 = 300e^{-kt}$. $\frac{\ln \frac{1}{100}}{-0.157} = t \Rightarrow t = 29.33\ldots$ time taken for tablet to dissolve to at least 1% 30 second		1 1 * 29 sec incorrect 1
M1 b) In Δ ADC, Δ BEC $\angle ADC = \angle BEC$ (given) $\angle DCA = \angle ECB$ (common) $\therefore \Delta ADC \parallel\!\mid \Delta BEC$ (equiangular) $\therefore$ ratios of corresponding sides in similar Δ's are in same ratio $\frac{CE}{CD} = \frac{BE}{AD}$ $CE = \frac{88}{9}$ or $9\frac{7}{9}$ or $9.\dot{7}$	 1 1	 1

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MATHEMATICS: Question……..

Suggested Solutions	Marks Awarded	Marker's Comments

M2

Since $BE \perp AC$ and $AD \perp BC$

$AC = \sqrt{9^2+8^2} = \sqrt{145}$ —— 1

Sine rule.

$$\frac{\sqrt{145}}{\sin 90} = \frac{9}{\sin \angle ACD}$$

$$\sin \angle ACD = \frac{9}{\sqrt{145}} = 48 \cdot 36\ldots^\circ$$ —— 1

$$\tan 48 \cdot 36 \ldots = \frac{11}{EC}$$ —— 1

$$EC = 9.\dot{7}$$

M3

let $\angle ACD = \angle ECD = \theta$ —— 1

In ΔACD, $\tan\theta = \frac{AD}{CD} = \frac{9}{8}$ ($\angle ADC$ is 90°, $AD \perp BC$) —— 1

Similarly in ΔBCE, $\tan\theta = \frac{BE}{CE} = \frac{11}{CE}$ ($\angle BEC = 90^\circ$ as $BE \perp AC$)

$$\frac{9}{8} = \frac{11}{CE}$$

$$CE = \frac{11 \times 8}{9} = \frac{88}{9}.$$ —— 1

c) i) $x = 2\cos t - t$; $t \geq 0$

$x(0) = 2\cos(0) - 0$

$\therefore$ 2 metres to right of origin —— 1

must mention right coz ∉ stated in questi

3/4

MATHEMATICS: Question……..

Suggested Solutions	Marks Awarded	Marker's Comments

ii) $v = \frac{dx}{dt} = -2\sin t - 1$ —— 1

iii) $v(0) = -2(0) - 1$

$= -1$ m/s.

Particle moving in $-$ve direction towards origin initially — 1

must mention towards origin as stated in question

iv) $v = 0$

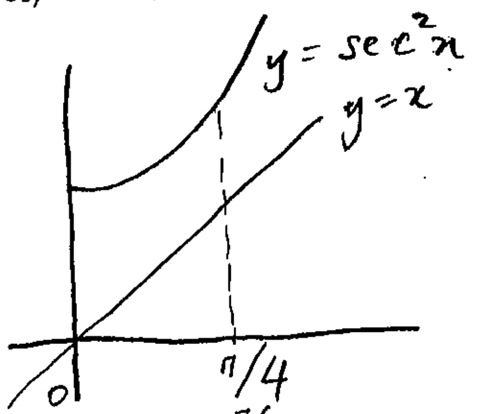
1 — $-2\sin t - 1 = 0$

$\sin t = -\frac{1}{2}$

$t = \frac{7\pi}{6}$ secs. or $\frac{11\pi}{6}$ 210° 330°

but 1st comes to rest is $\frac{7\pi}{6}$ seconds. —— 1

d)



Area $= \int_0^{\pi/4} \sec^2 x - x$ —— 1

$= \left[\tan x - \frac{x^2}{2}\right]_0^{\pi/4}$ —— 1

MATHEMATICS: Question......14		4/4
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
$\tan\frac{\pi}{4} - \frac{\left(\frac{\pi}{4}\right)^2}{2} - 0$		
$= 1 - \frac{\pi^2}{32}\ u^2.$	1	

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MATHEMATICS: Question 15 P1

Suggested Solutions	Marks	Marker's Comments
(a)(i) $\alpha+\beta = \frac{-b}{a} = \frac{-4}{1} = -4$ $\quad \alpha\beta = \frac{c}{a} = \frac{1}{1} = 1$.	1	
(ii) $(\alpha-\beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$ $= \alpha^2 + 2\alpha\beta + \beta^2 - 4\alpha\beta$ $= (\alpha+\beta)^2 - 4\alpha\beta$	1	
$= (-4)^2 - 4(1)$ $= 16 - 4$		Question well done.
$= 12.$	1	3
(b) $P = \$500000$ $\quad r = \frac{.09}{12.} = 0.0075$ $\quad n = 25\text{yrs} = 300\text{months}.$		
(i) Let amount owing after 1st month $= A_1$		
$A_1 = 500000(1+.0075) - M$ $= 500000(1.0075) - M$	1	You must derive A_3 by starting with A_1 & A_2.
$A_2 = (500000(1.0075) - M)1.0075 - M$ $= 500000(1.0075)^2 - M(1.0075) - M$		No marks for just stating A_3 – the answer is given
$A_3 = [500000(1.0075)^2 - M(1.0075) - M]1.0075 - M$ $= 500000(1.0075)^3 - M(1.0075)^2 - M(1.0075) - M$ $= 500000(1.0075)^3 - M(1 + 1.0075 + 1.0075^2)$	1	

Suggested Solutions	Marks	Marker's Comments
(ii) $n = 300$ months		
$A_n = 500000(1\cdot0075)^n - M(1 + 1\cdot0075 + 1\cdot0075^2 + \dots + 1\cdot0075^{n-1})$		
This is a GP where $n=300$, $r = 1\cdot0075$		
ie $S_{300} = \dfrac{1\cdot0075^{300}-1}{1\cdot0075-1}$	1	You need to show some calculation for this.
$A_{300} = 500000(1\cdot0075)^{300} - M\left(\dfrac{1\cdot0075^{300}-1}{\cdot0075}\right)$		
The loan is paid off when $A_{300}=0$		
ie $500000(1\cdot0075)^{300} - M\left(\dfrac{1\cdot0075^{300}-1}{\cdot0075}\right)=0$		
$M\left(\dfrac{1\cdot0075^{300}-1}{\cdot0075}\right) = 500000(1\cdot0075)^{300}$		
$M = \dfrac{500000(1\cdot0075)^{300}}{\left(\dfrac{1\cdot0075^{300}-1}{\cdot0075}\right)}$	1	
$= \$4195\cdot98$ (nearest cent) or $\$4196$ (nearest dollar).		
(iii) Total money paid on the loan over 300 months at \$4196 per month. is $300 \times \$4196 = \$1258794\cdot55$		
∴ Interest = Total paid − loan amount		
$= \$1258794\cdot55 - 500000$		
$= \$758794\cdot55$	1	

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Suggested Solutions	Marks	Marker's Comments
(c) $t=0$ $V = 16000$		
(i) $\dfrac{dV}{dt} = -40(30-t)$ $V = \int \dfrac{dV}{dt}\, dt$ $= \int -40(30-t)\, dt$ $= -40\int 30-t\, dt$ $= -40\left(30t - \dfrac{t^2}{2}\right) + c$ $V = -1200t + 20t^2 + c$	1	
When $t=0$ $V = 16000$ $\therefore 16000 = 0 + 0 + c$ $\therefore c = 16000$ $\therefore V(t) = -1200t + 20t^2 + 16000$	1	
(ii) Tank is empty when $V=0$ $\therefore -1200t + 20t^2 + 16000 = 0$ $20t^2 - 1200t + 16000 = 0$ $t^2 - 60t + 800 = 0$ $\quad \begin{matrix} -40 \\ -20 \end{matrix}$ $(t-40)(t-20) = 0$ $\therefore t = 20$ or 40 $\therefore$ The tank is empty after 20 mins. (Note: already empty before $t=40$ so only use $t=20$)	1	

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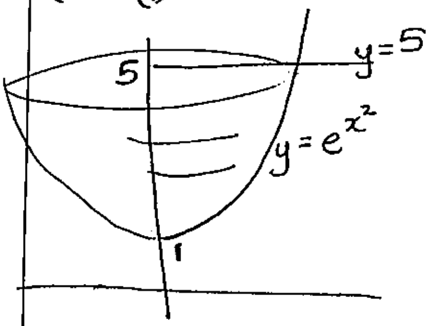
Suggested Solutions	Marks	Marker's Comments

(d) (i)



$$V=\pi\int_1^5 x^2\,dy$$ ①

Since $y=e^{x^2}$

$\log_e y = x^2$

$\therefore x^2=\log_e y$ ①

$$V=\pi\int_1^5 x^2\,dy$$

$$=\pi\int_1^5 \log_e y\,dy$$

← No mark for this because answer was given.

(ii)

y	1	3	5
$\ln y$	0	1.099	1.609

①

$$V=\pi\left[\frac{5-1}{6}\left(f(1)+4f(3)+f(5)\right)\right]$$

$$=\pi\left(\frac{2}{3}\left(0+4\times1.099+1.609\right)\right)$$ ①

$$=\pi\times4.00333$$

$$=12.577\ u^3\quad(3\text{dp})$$

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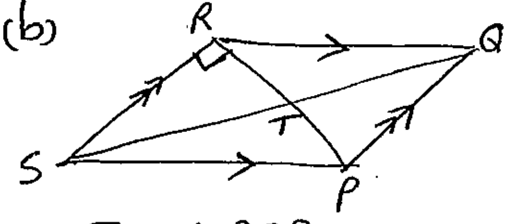
MATHEMATICS: Question 16		P1
Suggested Solutions	Marks	Marker's Comments
(a) $\log_4(x+1) = \log_2 x \quad x>0$ $\dfrac{\log_2(x+1)}{\log_2 4} = \log_2 x$ $\dfrac{\log_2(x+1)}{\log_2 2^2} = \log_2 x$ $\dfrac{\log_2(x+1)}{2\log_2 2} = \log_2 x$ $\log_2(x+1) = 2\log_2 x$ $\log_2(x+1) = \log_2 x^2$ $\therefore\ x^2 = x+1$ $x^2 - x - 1 = 0$ $x = \dfrac{1 \pm \sqrt{1-4\times -1}}{2}$ $x = \dfrac{1\pm\sqrt{5}}{2}$ but $x>0$ $\therefore\ x = \dfrac{1+\sqrt{5}}{2}$ is only solution. $= 1.618$ (3dp) $x>0$.	 1 1	

Suggested Solutions	Marks	Marker's Comments

(b)



In ΔPSR

$PS^2 = RS^2 + RP^2$ (A) (Pythagoras' Theorem)

In ΔRST

$ST^2 = RS^2 + RT^2$ (Pythagoras' Theorem)

$\therefore RS^2 = ST^2 - RT^2$

$\therefore RS^2 = ST^2 - PT^2$ (B) ($RT = PT$, diagonals of a parallelogram bisect each other).

Subst (B) into (A)

$PS^2 = ST^2 - PT^2 + RP^2$

Note $RP = 2PT$ (diagonals of rhombus bisect each other).

$PS^2 = ST^2 - PT^2 + (2PT)^2$

$PS^2 = ST^2 - PT^2 + 4PT^2$

$PS^2 = ST^2 + 3PT^2$

(c)(i) $l = r\theta$ — 1

$AP = r2\theta$

If travel from $AP = r2\theta$ at 4 kmh

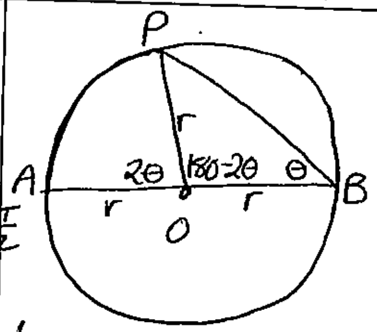
$\therefore time_{AP} = \dfrac{r2\theta}{4}$ — 1

$t_1 = \dfrac{r\theta}{2}$ ← Answer was given.

MATHEMATICS: Question 16. p3.

Suggested Solutions | **Marks** | **Marker's Comments**

(c) (ii) to get from PB rowing.



$$\text{time} = \frac{\text{distance } PB}{\text{speed}}$$

Using the $\underline{\cos}$ $\underline{\text{Rule}}$.

$$d_{PB}^2 = r^2 + r^2 - 2r \cdot r \cdot \cos(180 - 2\theta) \quad 0 \leq \theta \leq \tfrac{\pi}{2}$$

$$= 2r^2 + 2r^2 \cos 2\theta$$ (1)

$\cos 2\theta$ in the second quadrant

Since $\cos 2\theta = 2\cos^2\theta - 1$

$$d_{PB}^2 = 2r^2 + 2r^2(2\cos^2\theta - 1)$$

$$= 2r^2 + 4r^2\cos^2\theta - 2r^2$$ (1)

$$d_{PB}^2 = 4r^2\cos^2\theta$$

$$d_{PB} = \pm\sqrt{4r^2\cos^2\theta} \qquad d_{PB} > 0$$

$$\therefore d_{PB} = 2r\cos\theta$$ (1)

$\therefore$ time taken to row from P to B

$$= \frac{2r\cos\theta}{2}$$

$$t_2 = r\cos\theta, \quad 0 \leqslant \theta \leqslant \tfrac{\pi}{2}$$

Once again because the result is given you need to

NOTE: $\boxed{\text{Alternatively}}$ use the Sine Rule.

Better alternative

$$\frac{d_{PB}}{\sin(180 - 2\theta)} = \frac{r}{\sin\theta}$$

$$\frac{d_{PB}}{\sin 2\theta} = \frac{r}{\sin\theta}$$

$$d_{PB} = r\,\frac{\sin 2\theta}{\sin\theta}$$

$$= r\,\frac{2\sin\theta\cos\theta}{\sin\theta}$$

$$= 2r\cos\theta$$

$$\therefore \text{time to row} = \frac{2r\cos\theta}{2} = r\cos\theta.$$

MATHEMATICS: Question 16 P4

Suggested Solutions	Marks	Marker's Comments

(ii) Maximum time taken for journey

Time $= t_1 + t_2$

$t = \frac{r\theta}{2} + r\cos\theta$ $\quad$ r is constant, θ is variable.

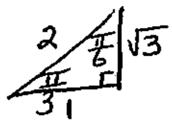
$\frac{dt}{d\theta} = \frac{r}{2} - r\sin\theta$

(iii) (α) Max/Min values occur when $\frac{dt}{d\theta} = 0$

ie $\frac{r}{2} - r\sin\theta = 0$

$r\sin\theta = \frac{r}{2}$

$\sin\theta = \frac{1}{2}$



$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}, -$

$= \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $0 \leq \theta \leq \frac{\pi}{2}$, $\theta \neq \frac{5\pi}{6}$ $\quad$ (1)

To test nature of stationary point

$\frac{d^2t}{d\theta^2} = -r\cos\theta$

When $\theta = \frac{\pi}{6}$ $\quad$ $\frac{d^2t}{d\theta^2} = -r\cos\frac{\pi}{6}$

$= -r\frac{\sqrt{3}}{2}$

< 0

∴ concave down $\quad$ (1)

∴ max time when $\theta = \frac{\pi}{6}$.

∴ Max time of journey $= \frac{r\frac{\pi}{6}}{2} + r\cos\frac{\pi}{6}$

$= \frac{r\pi}{12} + \frac{r\sqrt{3}}{2}$ $\quad$ (1)

MATHEMATICS: Question 16

P5

Suggested Solutions	Marks	Marker's Comments
Max time $= \frac{r\pi}{12} + \frac{r\sqrt{3}}{2}$ hours.		
(iii) (β) Since there are no other stationary points $0 \le \theta \le \frac{\pi}{2}$, then test the endpoints for a minimum journey.		
When $\underline{\theta = 0}$ the person has rowed across the lake.		
ie $t = \frac{r}{2} \times 0 + r\cos\theta$ $= r$		
<u>OR</u> of course if they have rowed the whole way the distance is $2r$ $\therefore$ time $= \frac{2r}{2}$ $= r$.		
When $\theta = \frac{\pi}{2}$ $t = \frac{r}{2}\left(\frac{\pi}{2}\right) + r\cos\left(\frac{\pi}{2}\right)$ $t = \frac{r\pi}{4}$ $\quad \frac{r\pi}{4} < r$	1	Think! – Technically the question stated a combination of walking & rowing. So the minimum time would be walk the whole way except for the last few steps & row a tiny bit!!
Note: this is the minimum time when $t = r\frac{\pi}{4}$. Note: $C = \frac{1}{2}2\pi r$ $= \pi r$ time $= \frac{\pi r}{4}$.		
It is the time if the person walked the full distance.	1	Some discussion